

**Winter Examination – 2022**

**Course:** - B. Tech.

**Branch:** - Common for All branches

**Semester:-** III

**Subject Code & Name:** BTBS301

**Engineering Mathematics-III**

**Max. Marks:** - 60

**Date:** - 09/03/2023

**Duration:** - 3-Hrs

**Instructions to the Students:**

1. All the questions are compulsory.
2. The level of question/expected answer as per OBE or the Course Outcome (CO) on which the question is based is mentioned in () in front of the question.
3. Use of non-programmable scientific calculators is allowed.
4. Assume suitable data wherever necessary and mention it clearly.

	(Level/CO)	Marks
<b>Q. 1 Solve Any Three of the following.</b>		<b>12</b>
A) Find Laplace Transform of $e^{-3t} \sin^2 t$	L3/CO1	<b>4</b>
B) Find Laplace Transform of $f(t) = \begin{cases} 1 & 0 < t < 1 \\ 0 & 1 < t < 2 \end{cases}$	L3/CO1	<b>4</b>
where $f(t)$ is periodic function of period 2.		
C) Evaluate using Laplace Transform.: $\int_0^\infty \frac{\cos 4t - \cos 3t}{t} dt$	L3/CO1	<b>4</b>
D) Find Laplace Transform of $(1 + 2t - 3t^2 + 4t^3)H(t - 2)$	L3/CO1	<b>4</b>
 <b>Q2 Solve Any Three of the following.</b>		 <b>12</b>
A) Find the inverse Laplace transformation of the function. $\log \left( 1 + \frac{a^2}{s^2} \right)$	L3/CO2	<b>4</b>
B) By using convolution theorem find $L^{-1} \left[ \frac{s}{(s^2+4)(s^2+9)} \right]$	L3/CO2	<b>4</b>
C) Find the inverse Laplace transformation of the function. $\frac{5s^2-15s-11}{(s+1)(s-2)^2}$	L3/CO2	<b>4</b>
D) Solve using Laplace transformation		
$y'' + 3y' + 2y = t\delta(t - 1)$ for which $y(0) = y'(0) = 0$	L3/CO2	<b>4</b>

**Q.3 Solve Any Three of the following. (12)**

A) Using Parseval's identity prove that  $\int_0^{\infty} \frac{x^2}{(x^2+1)^2} dx = \frac{\pi}{4}$  L3/CO3 4

B) Find the Fourier transform of

$$f(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0 & , |x| > 1 \end{cases} \quad \text{L3/CO3} \quad 4$$

C) Find the Fourier Sine transform  $e^{-ax}$ ,  $a > 0$  L3/CO3 4

D) Find the Fourier cosine transform of the function  $f(y) = \begin{cases} \cos y, & 0 < y < a \\ 0, & y > a \end{cases}$  L3/CO3 4

**Q.4 Solve Any Three of the following. (12)**

A) Form the partial differential equation by eliminating arbitrary constants from L3/CO4 4

$$(x - a)^2 + (y - b)^2 = z^2 \cot^2 \alpha$$

B) Solve the Partial differential equation  $x(y - z)p + y(z - x)q = z(x - y)$  L3/CO4 4

C) Use the method of separation of variables to solve

$$\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u \quad \text{given that } u(x, 0) = 6e^{-3x} \quad \text{L3/CO4} \quad 4$$

D) A bar with insulated at its ends is initially at temperature  $0^\circ\text{C}$  throughout. The end  $x = 0$  is kept at  $0^\circ\text{C}$  for all times and the heat is suddenly applied so that  $\frac{\partial u}{\partial x} = 10$  at  $x = t$  for all time. Find the temperature function  $u(x, t)$  L3/CO4 4

**Q.5 Solve Any Three of the following. (12)**

A) Determine  $k$  such that the function  $f(z) = e^x \cos y + ie^x \sin ky$  is analytic. L3/CO5 4

B) Show that  $u = x^2 - y^2 - 2xy - 2x + 3y$  is a harmonic function and L3/CO5 4  
hence determine the analytic function  $f(z)$  in terms of  $z$ .

C) Determine the pole of the function  $f(z) = \frac{2z-1}{z(z+1)(z-3)}$  and also find the residue at each pole

& sum of all residues. L3/CO5 4

D) Evaluate L3/CO5 4

$$\oint_C \frac{\sin \pi z^2 + 2z}{(z-1)^2(z-2)} dz, \text{ Where } C \text{ is the circle } |z| = 4$$

\*\*\* End \*\*\*